Invitational World Youth Mathematics Intercity Competition 1999

Team Contest

1. (a) Decompose $9^8+7^6+5^4+3^2+1$ into prime factors.
   (b) Find two distinct prime factors of $2^{30}+3^{20}$.

2. The cards in a deck are numbered 1, 3, $\cdots$, $2n-1$. In the k-th step, $1 \le k \le n$, $2k-1$ cards from the top of the deck are transferred to the bottom one at a time. We want the new card on the top to be $2k-1$, which is then set aside. After n steps, the whole deck should be set aside in increasing order. How should the deck be stacked in order for this to happen, if
   (a) n=10;
   (b) n=30?

3. (a) Express 1 as a sum of trhe reciprocals of distinct integers, one of which is 5.
   (b) Express 1 as a sum of trhe reciprocals of distinct integers, one of which is 1999.

4. (a) Show how to dissect a square into 1999 squares which may have different sizes.
   (b) Dissect the first two shapes in the diagram below into the ten or fewer pieces which can be reassembled to form the third shape.

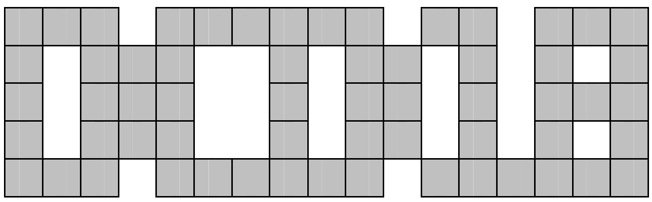

Figure (1)

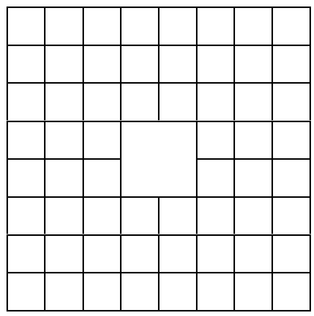

Figure (2)

5. The diagram below shows a blank 5×5 table. Each cell is to be filled in with one of the numbers 1, 2, 3, 4 and 5, so there is exactly one number of each kind in each row, each column and each of the two long diagonals. The score of a completed table is the sum of the numbers in the four shaded cells. What is the highest possible score of a completed table? ◦