Invitational World Youth Mathematics Intercity Competition 2001

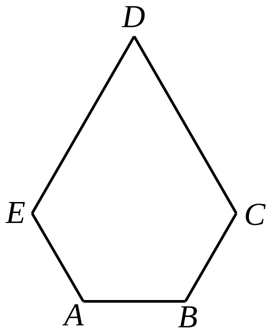
Individual Contest

Section A.

In this section, there are 10 questions. Fill in the correct answer in the space provided at the end of each question. Each correct answer is worth 6 points.

1. Find all integers n such that $1 + 2 + .. + n$ is equal to a 3-digit number with identical digits.

2. In a convex pentagon $ABCDE$, $\angle A = \angle B = 120°$, $EA = AB = BC = 2$, and $CD = DE = 4$. Find the area of the pentagon $ABCDE$.



3. If I place a 6 cm by 6 cm square on a triangle, I can cover up to 60% of the triangle. If I place the triangle on the square, I can cover up to $\frac{2}{3}$ of the square. What is the area of the triangle?

4. Find a set of four consecutive positive integers such that the smallest is a multiple of 5, the second is a multiple of 7, the third is a multiple of 9, and the largest is a multiple of 11.

5. Between 5 and 6 o'clock, a lady looked at her watch. She mistook the hour hand for the minute hand and vice versa. As a result, she thought the time was approximately 55 minutes earlier. Exactly how many minutes earlier was the mistaken time?

6. In triangle ABC, the incircle touches the sides BC, CA and AB at D, E and F respectively. If the radius of the incircle is 4 units and if BD, CE and AF are consecutive integers, find the length of the three sides of ABC.

7. Determine all primes p for which there exists at least one pair of integers x and y such that $p+1=2x^2$ and $p^2+1=2y^2$.

8. Find all real solutions of
$$\sqrt{3x^2-18x+52}+\sqrt{2x^2-12x+162}=\sqrt{-x^2+6x+280}.$$

9. Simplify $\sqrt{12-\sqrt{24}+\sqrt{39}-\sqrt{104}}-\sqrt{12+\sqrt{24}+\sqrt{39}+\sqrt{104}}$ into a single numerical value.

10. Let M = 1010101…01 where the digit 1 appears k times. Find the least value of k so that 1001001001001 divides M ?

Section B.

Answer the following 3 questions, and show your detailed solution in the space provided after each question. Each question is worth 20 points.

1. Given that a and b are unequal positive real numbers, let $A=\frac{a+b}{2}$ and $G=\sqrt{ab}$. Prove that the following inequality holds: $G<\frac{(a-b)^2}{8(A-G)}<A$.

2. Find the range of p such that the equation $3^{2x}-3^{x+1}=p$ has two different real positive roots.

3. The four vertices of a square lie on the perimeter of an acute scalene triangle, with one vertex on each of two sides and the other two vertices on the third side. If the square is be as large as possible, should the side of the triangle containing two vertices of the square be the longest, the shortest or neither? Justify your answer.