2009 Philippine Elementary Mathematics International Contest

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Individual Contest

Time limit: 90 minutes 2009/11/30

Instructions:

- Do not turn to the first page until you are told to do so.
- Write down your name, your contestant number and your team's name on the answer sheet.
- Write down all answers on the answer sheet. Only Arabic NUMERICAL answers are needed.
- Answer all 15 problems. Each problem is worth 10 points and the total is 150 points. For problems involving more than one answer, full credit will be given only if ALL answers are correct, no partial credit will be given.
- Diagrams shown may not be drawn to scale.
- No calculator or calculating device is allowed.
- Answer the problems with pencil, blue or black ball pen.
- All papers shall be collected at the end of this test.

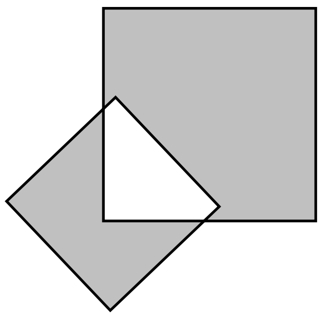
English Version

1. Find the smallest positive integer whose product after multiplication by 543 ends in 2009.

2. Linda was delighted on her tenth birthday, 13 July 1991 (13/7/91), when she realized that the product of the day of the month together with the month in the year was equal to the year in the century: $13\times7 = 91$. She started thinking about other occasions in the century when such an event might occur, and imagine her surprise when she realized that the numbers in her two younger brothers' tenth birthdays would also have a similar relationship. Given that the birthdays of the two boys are on consecutive days, when was Linda's youngest brother born?

3. Philip arranged the number 1, 2, 3, ... , 11, 12 into six pairs so that the sum of the numbers in any pair is prime and no two of these primes are equal. Find the largest of these primes.

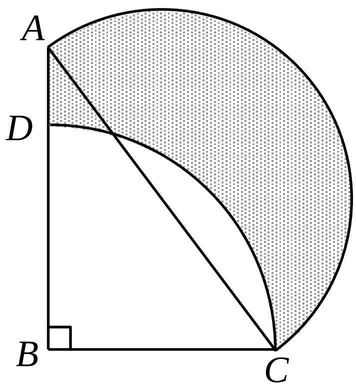
4. In the figure, $\frac{3}{4}$ of the larger square is shaded and $\frac{5}{7}$ of the smaller square is shaded. What is the ratio of the shaded area of the larger square to the shaded area of the smaller square?



5. Observe the sequence 1, 1, 2, 3, 5, 8, 13, … . Starting from the third number, each number is the sum of the two previous numbers. What is the remainder when the 2009^{th} number in this sequence is divided by 8?

6. Ampang Street has no more than 15 houses, numbered 1, 2, 3 and so on. Mrs. Lau lives in one of the houses, but not in the first house. The product of all the house numbers before Mrs. Lau's house, is the same as that of the house numbers after her house. How many houses are on Ampang Street?

7. In the given figure, ABC is a right-angled triangle, where $\angle B = 90°$, $BC = 42$ cm and $AB = 56$ cm. A semicircle with AC as a diameter and a quarter-circle with BC as radius are drawn. Find the area of the shaded portion, in cm^2. (Use $\pi = \frac{22}{7}$)

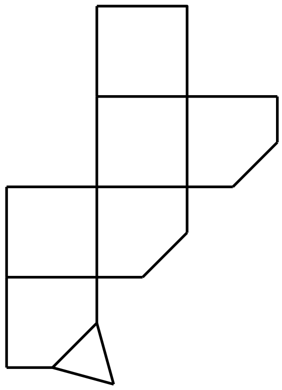


8. A number consists of three different digits. If the difference between the largest and the smallest numbers obtained by rearranging these three digits is equal to the original number, what is the original three-digit number?

9. The last 3 digits of some perfect squares are the same and non-zero. What is the smallest possible value of such a perfect square?

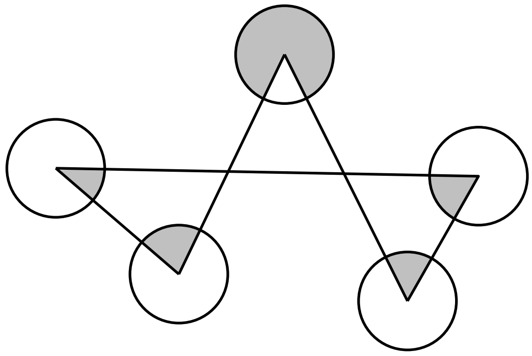
10. Lynn is walking from town A to town B, and Mike is riding a bike from town B to town A along the same road. They started out at the same time and met 1 hour after. When Mike reaches town A, he turns around immediately. Forty minutes after they first met, he catches up with Lynn, still on her way to town B. When Mike reaches town B, he turns around immediately. Find the ratio of the distances between their third meeting point and the towns A and B.

11. The figure shows the net of a polyhedron. How many edges does this polyhedron have?



12. In the figure, the centers of the five circles, of same radius 1 cm, are the vertices of the triangles. What is the total area, in cm^2, of the shaded regions?

    (Use $\pi = \frac{22}{7}$)



13. There are 10 steps from the ground level to the top of a platform. The 6^{th} step is under repair and can only be crossed over but not stepped on. Michael walks up the steps with one or two steps only at a time. How many different ways can he use to walk up to the top of the platform?

14. For four different positive integers a, b, c and d, where $a < b < c < d$, if the product $(d - c) \times (c - b) \times (b - a)$ is divisible by 2009, then we call this group of four integers a "friendly group". How many "friendly groups" are there from 1 to 60?

15. The figure shows five circles A, B, C, D and E. They are to be painted, each in one color. Two circles joined by a line segment must have different colors. If five colors are available, how many different ways of painting are there?